## Newton's 2nd law/ demonstration track with Cobra4

(Item No.: p2130360)

## Curricular Relevance

| Area of Expertise: <br> Physics | Education Level: <br> University | Topic: <br> Mechanics | Subtopic: <br> Motion in one <br> Dimension |
| :--- | :--- | :--- | :--- |
| Difficulty |  |  |  |

## Keywords: <br> Linear motion, Velocity, Acceleration, Conservation of energy

## Overview

## Short description

## Principle

The distance-time law, the velocity-time law, and the relationship between mass, acceleration and force are determined with the aid of the demonstration track for uniformly accelerated motion in a straight line.


Fig. 1: Experimental set-up.

## Equipment

| Position No. | Material | Order No. | Quantity |
| :--- | :--- | :--- | :--- |
| 1 | Cobra4 Wireless/USB-Link | $12601-10$ | 1 |
| 2 | Cobra4 Sensor-Unit Timer/Counter | $12651-00$ | 1 |
| 3 | Light barrier, compact | $11207-20$ | 1 |
| 4 | Demonstration track, aluminium | $11305-00$ | 1 |
| 5 | Holder for pulley | $11305-11$ | 1 |
| 6 | End holder for demonstration track | $11305-12$ | 1 |
| 7 | Cart, low friction sapphire bearings | $11306-00$ | 1 |
| 8 | Starter system for demonstration track | $11309-00$ | 1 |
| 9 | Pulley,movable,dia.40mm,w.hook | $03970-00$ | 1 |
| 10 | Tube with plug | $11202-05$ | 1 |
| 11 | Needle with plug | $11202-06$ | 1 |
| 12 | Magnet w.plug f.starter system | $11202-14$ | 1 |
| 13 | Plasticine, 10 sticks | $03935-03$ | 1 |
| 14 | Silk thread, 200 m | $02412-00$ | 1 |
| 15 | Weight holder 1 g | $02407-00$ | 1 |
| 16 | Slotted weight, 1 g, natur.colour | $03916-00$ | 20 |
| 17 | Slotted weight, 10 g, black | $02205-01$ | 4 |
| 18 | Slotted weight, 10 g, silver | $02205-02$ | 4 |
| 19 | Slotted weight, 50 g, black | $02206-01$ | 2 |
| 20 | Slotted weight, 50 g, silver | $02206-02$ | 2 |
| 21 | Connecting cord, 32 A, 1000 mm, red | $07363-01$ | 1 |
| 22 | Connecting cord, 32 A, 1000 mm, blue | $07363-04$ | 1 |
| 23 | Portable Balance, OHAUS CS2000 | $48917-93$ | 1 |
| 24 | Software measure for Cobra4 | $14550-61$ | 1 |
| Additionally |  |  |  |
| required |  | PC with USB interface, Windows XP or <br> higher |  |
|  |  |  |  |

## Tasks

Determination of:

1. Distance travelled as a function of time
2. Velocity as a function of time
3. Acceleration as a function of the accelerated mass
4. Acceleration as a function of force.

## Notes

It is necessary to ensure that when mass $m_{1}$ falls, the thread rolls over the incremental wheel and is so brought to turn. It must also be ensured that mass $m_{1}$ does not oscillate before and during measurement and can drop freely to the floor below without touching the edge of the table.

The plasticine must be brought back to the initial state between measurements so that the impact of the cart is cushioned to the best possible extent.
After each experiment, check that all slotted weights of mass $m_{1}$ are still on the weight holder.
Ensure that the thread runs parallel to the track and is taut before and during measurement.

## Setup and Procedure

## Set-up

- As shown in Figure 1.
- Place the track on a level surface. If necessary use the three adjusting screws to align it horizontally.
- Screw the starting device so on the end of the track that when the plunger is triggered it is drawn back into the starting device without imparting any impact.
- Connect the upper starting device connector to the "Mass" input and the red-sheathed connector to the "Start" input of the Timer-Counter Sensor-Unit.
- Screw the end holder onto the other end of the track and insert the tube filled with plasticine.
- Use the deflection pulley holder to fix the light barrier to the track end holder.
- Plug the adapter on the light barrier and connect it to the Timer-Counter Sensor-Unit.
- Fit the incremental wheel in the light barrier.
- Put the cart with screwed-on rod for holding weights on the track. Plug the holding magnet in the side of the cart which faces the starting device.
- Use the needle with plug to fix the end of the thread to the cart in the direction of cart travel as fol-lows: Plug the thread to the cart in the upper hole and the needle with plug through the side hole so that the thread is clamped in position.
- Bring the cart to the starting position so that both the starter and the holding magnet make contact.
- Lead the thread to run in the direction of travel and over the incremental wheel. Cut the thread to be long enough to be tied to the weight holder so that his can then hang freely.
- Tie the free end of the thread to the weight holder.


## Procedure

- Start the PC and Windows.
- Plug Cobra4 Wireless/USB-Link via USB-cable in the USB-port of the PC.
- Start the measure software package on the PC.
- Switch on the Cobra4 Wireless/USB-Link with plugged on Timer-Counter Sensor-Unit.
- Use the compact balance to measure the mass of the cart $m_{2}$ with attached rod for holding additional weights, holding magnet and needle with plug.
- Load the experiment in measure (Experiment > Open experiment). All pre-settings which are necessary for the recording of measured values are now started.
- Press the metal plunger in the starting device so that the plunger is flush with the cylindrical opening into which it will be drawn. This hold ensures release of the cart without a starting impact.
- Start the recording of measured values in measure
- Trigger the starting device to release the cart so that it rolls along the track.
- Stop measurement with the box ■ before driving mass $m_{1}$ reaches the floor. Transfer the measured data to measure.
- The measurement can now be repeated to check the reproducibility of individual measurements and to reduce the deviation of the final result from the literature value. You can then determine the mean result from the values obtained in the individual measurements.
- In Part A, the driving mass $m_{1}=10 \mathrm{~g}$ is to remain constant and 9 slotted weights of 1 g mass are to be fitted on. The first measurement is to be made without additional mass $m_{z}$. In subsequent measurements the mass of the cart is to be increased in 5 g steps up to 50 g and measurement made at each weight increase step. Record the average value of the acceleration and the appropriate values for the mass of the cart and the additional mass.
- In Part B the total mass is to remain constant. An accelerating mass of 5 g is to be used to start with, but remember here that the weight holder itself has a weight of 1 g , so that only 4 slotted weights of 1 g are to be hung on. An additional weight of 5 g must also be brought onto the cart. The first measurement can now be started. Following this, make measurements stepwise by transferring one 1 g slotted weight from $m_{1}$ to $m_{2}$ at each step.
- Prior to each new measurement series, i.e. on each variation of mass, use the balance to determine the mass of the cart with additional weights ( $m_{2}+m_{z}$ ) as well as mass of the driving mass $m_{1}$
- To determine the acceleration as a function of the mass, increase the mass of the cart progressively by 20 g increments, and measure the instantaneous velocity at a predetermined position. In determining the acceleration as a function of force, the total mass remains constant. Successively transfer 2 g from the cart to the weight holder and measure the instantaneous velocity at a fixed position. The accelerated mass must not exceed 20 g . Before beginning with the measurements, it is advisable to check the track's adjustment.


## Remarks

If the values ( 50 ms ) in the "General configuration" tab are too high or too low, noisy or non-uniform measurements can occur. In this case, adjust the measurement sampling rate appropriately. At excessively low velocities and exceedingly high sampling rates (short ms times) the velocity $0 \mathrm{~m} / \mathrm{s}$ can be intermittently measured at irregular intervals. In this case increase the sampling time.

## Theory and Evaluation

Newton's equation of motion for a mass point of mass $m$ to which a force is applied is given by the following:

$$
m \cdot \vec{a}=\vec{F}
$$

where

$$
\vec{a}=\frac{d^{2} \vec{r}}{d t^{2}}
$$

is the acceleration.
The velocity $v$ obtained by application of a constant force is given as a function of the time $t$ by the expression

$$
\vec{v}(t)=\frac{\vec{F}}{m} t
$$

for

$$
\vec{v}(0)=0
$$

Assuming that

$$
\vec{v}(0)=0 ; \quad \vec{r}(0)=0
$$

the position of $\vec{F}$ of the mass point is

$$
\begin{equation*}
\vec{r}(t)=\frac{1}{2} \frac{\vec{F}}{m} \cdot t^{2} \tag{0}
\end{equation*}
$$

In the present case the motion is one dimensional and the force produce by a weight of $m_{1}$ is

$$
|\vec{F}|=m_{1} \cdot|\vec{g}|=m_{1} \cdot g
$$

where $g$ is the acceleration of gravity. If the total mass of the cart is $m_{2}$ the equation of motion is given by

$$
\begin{equation*}
\left(m_{2}+m_{1}\right) \cdot|\vec{a}|=m_{1} \cdot g \tag{1}
\end{equation*}
$$

The velocity is

$$
\begin{equation*}
|\vec{v}(t)|=v=\frac{m_{1} \cdot g}{m_{1}+m_{2}} \cdot t \tag{2}
\end{equation*}
$$

and the position is

$$
\begin{equation*}
|\vec{r}(t)|=s(t)=\frac{1}{2} \frac{m_{1} \cdot g}{m_{1}+m_{2}} \cdot t^{2} \tag{3}
\end{equation*}
$$

Typical measurement data are displayed in a $v-t$ (or $s-t$ ) diagram (Fig. 2). In addition to the measured points of interest (the rising branch of the $v(t)$ curve), the collision phase of the glider with the stopper is also recorded. These latter measured points must be deleted before continuing with the evaluation. At the left image margin, at low velocities, the velocities have been only inadequately registered due to the slow rotation of the light barrier's wheel and show a nearly constant velocity course. Fig. 3 shows the velocity-time curve, a straight line, which conforms to the relationship $v=a \cdot t$. A regression line has been fitted to the measured points; the slope $m$ supplies the acceleration $a$, in this case $0.229 \mathrm{~m} / \mathrm{s}^{2}$.


Fig. 2: Measurement data before deletion of the unnecessary measured points.


Fig. 3: Velocity-time law with linear regression curve.
The time course of acceleration $a(t)$ is shown in Fig. 4. Initially, the acceleration increases. This is due to the light barrier's measurement principle. The light beam interruptions by the spoked wheel are counted for equal time intervals, in this case 50 ms . At very low velocities even with accelerated movement only one interruption per time interval occurs, i. e. the velocity is measured as a constant value. Subsequently, measured points of constant acceleration are recorded. Ultimately, the acceleration again decreases, i.e. when the glider reaches the end of the track or the stop. If the "Measure" icon is chosen, a horizontal cursor line can be positioned. It approximately passes through the centres of the points that represent constant acceleration. The measured average acceleration value agrees well with the value determined using regression in Fig. 3. Now the acceleration of gravity can be calculated. The following relationship applies (see equation (1)).

$$
\begin{gathered}
\left(m_{1}+m_{2}\right) a=m_{2} g \\
\quad \text { and thus } \\
\left(m_{1}+m_{2}\right) a / m_{2}=g
\end{gathered}
$$

Since $m_{1}$ (the mass of the cart) and $m_{2}$ (the accelerated mass) are known (their weights can be checked), the acceleration of gravity $g$ can be calculated when $a$ has been measured. For a typical measurement, where $m_{1}=200.2 \mathrm{~g}, m_{2}=5 \mathrm{~g}$ and $a=0.229 \mathrm{~m} / \mathrm{s}^{2}$ are measured, it follows that $g=9.4 \mathrm{~m} / \mathrm{s}^{2}$. According to equation (3) the curve of the path-time law exhibits a parabolic course (Fig. 5). This fact can be verified as follows: The time axis is squared to obtain a linearized curve course (Fig. 6). Using the Analysis / Channel Modification tool, the operation $\mathrm{f}:=\mathrm{t} * \mathrm{t}$ is performed (Fig.7).


Fig. 4: Acceleration-time law.


Fig. 5: Path-time law.


Fig. 6: Path-time law with the square of time on $x$ axis.


Fig. 7: Kinetic energy.
The regression line in Fig. 6 proves that the curve course is now linear and thus also the original quadratic dependence of the path on the time. In a very similar manner, other measured channels can also be mathematically transformed. To verify the energy balance, the kinetic and potential energy has to be calculated and displayed.
Kinetic energy (Fig. 7): $E_{\text {kin }}(t)=0.5 \cdot\left(m_{1}+m_{2}\right) v^{2}$, where $m_{1}+m_{2}=205.2 \mathrm{~g}$.
Conversion using: Analysis / Channel modification / Operation
$\mathrm{f}:=0.5 * 205.2 * \mathrm{x} * \mathrm{x}$, where $x=v(t)$.
Potential energy (Fig. 8): $E_{\text {pot }}(t)=m_{2} g(h-s(t))$, where $h=0.932 \mathrm{~m}$.
Conversion using: Analysis / Channel modification / Operation
$\mathrm{f}:=5 * 9.81 *(0.932-\mathrm{x})$, wherex $=s(t)$.
The law of conservation on energy states that the sum of kinetic and potential energy in this closed system must be constant. This statement can easily be checked by the addition of potential and kinetic energy (Fig. 9).


Fig. 8: Potential energy.


Fig. 9: Sum of the respective kinetic and potential energies.

